



TEST OF MATHEMATICS  
FOR UNIVERSITY ADMISSION

D513/01

PAPER 1

November 2021

75 minutes

Additional materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 20 questions. Each question is worth one mark.

You can use the question paper for rough working or notes, but **no extra paper** is allowed.

You **must** complete the answer sheet within the time limit.

Calculators and dictionaries are NOT permitted.

There is no formulae booklet for this test.

**Please wait to be told you may begin before turning this page.**

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**1** Two circles have the same radius.

The centre of one circle is  $(-2, 1)$ .

The centre of the other circle is  $(3, -2)$ .

The circles intersect at two distinct points.

What is the equation of the straight line through the two points at which the circles intersect?

**A**  $3x - 5y = 4$

**B**  $3x + 5y = -1$

**C**  $5x - 3y = -4$

**D**  $5x - 3y = -1$

**E**  $5x - 3y = 1$

**F**  $5x - 3y = 4$

**G**  $5x + 3y = 1$

- 2 The curve  $y = x^3 - 6x + 3$  has turning points at  $x = \alpha$  and  $x = \beta$ , where  $\beta > \alpha$ .

Find

$$\int_{\alpha}^{\beta} x^3 - 6x + 3 \, dx$$

- A  $-8\sqrt{2}$
- B  $-10$
- C  $-10 + 6\sqrt{2}$
- D  $0$
- E  $12 - 8\sqrt{2}$
- F  $6\sqrt{2}$
- G  $12$

- 3** An arithmetic progression and a convergent geometric progression each have first term  $\frac{1}{2}$

The sum of the second terms of the two progressions is  $\frac{1}{2}$

The sum of the third terms of the two progressions is  $\frac{1}{8}$

What is the sum to infinity of the geometric progression?

**A**  $-2$

**B**  $-1$

**C**  $-\frac{1}{2}$

**D**  $-\frac{1}{3}$

**E**  $\frac{1}{3}$

**F**  $\frac{1}{2}$

**G**  $1$

**H**  $2$

4 Find the minimum value of the function

$$2^{2x} - 2^{x+3} + 4$$

**A** -16

**B** -12

**C** -8

**D** 0

**E** 4

**F** 20

5 The function  $f$  is such that

$$f(mn) = \begin{cases} f(m)f(n) & \text{if } mn \text{ is a multiple of 3} \\ mn & \text{if } mn \text{ is not a multiple of 3} \end{cases}$$

for all positive integers  $m$  and  $n$ .

Given that  $f(9) + f(16) - f(24) = 0$ , what is the value of  $f(3)$ ?

- A  $\frac{8}{3}$
- B  $2\sqrt{2}$
- C 3
- D  $\frac{16}{5}$
- E  $3\sqrt{2}$
- F 4

**6** The function  $f$  is given by

$$f(x) = \frac{\cos x + 3}{7 + 5 \cos x - \sin^2 x}$$

Find the positive difference between the maximum and the minimum values of  $f(x)$ .

**A** 0

**B**  $\frac{1}{3}$

**C**  $\frac{1}{2}$

**D**  $\frac{2}{3}$

**E** 1

**F** 2



- 7 The function  $f$  is such that  $f(0) = 0$ , and  $xf(x) > 0$  for all non-zero values of  $x$ .

It is given that

$$\int_{-2}^2 f(x) \, dx = 4$$

and

$$\int_{-2}^2 |f(x)| \, dx = 8$$

Evaluate

$$\int_{-2}^0 f(|x|) \, dx$$

- A -8
- B -6
- C -4
- D -2
- E 2
- F 4
- G 6
- H 8

**8** The line  $y = 2x + 3$  meets the curve  $y = x^2 + bx + c$  at exactly one point.

The line  $y = 4x - 2$  also meets the curve  $y = x^2 + bx + c$  at exactly one point.

What is the value of  $b - c$ ?

**A**  $-9$

**B**  $-5.5$

**C**  $-1$

**D**  $5$

**E**  $6$

**F**  $14$

9 Find the area enclosed by the graph of

$$|x| + |y| = 1$$

A  $\frac{1}{2}$

B 1

C 2

D 4

E  $\frac{1}{2}\sqrt{2}$

F  $\sqrt{2}$

G  $2\sqrt{2}$

10 Use the trapezium rule with 3 strips to estimate

$$\int_{\frac{1}{2}}^2 2 \log_{10} x \, dx$$

A  $\log_{10} \frac{\sqrt{6}}{2}$

B  $\log_{10} \frac{3}{2}$

C  $\log_{10} \frac{9}{4}$

D  $\log_{10} 3$

E  $\log_{10} \frac{81}{16}$

F  $\log_{10} \frac{\sqrt{23}}{2}$

11 The function  $f$  is given by

$$f(x) = x^{\frac{1}{7}}(x^2 - x + 1)$$

Find the fraction of the interval  $0 < x < 1$  for which  $f(x)$  is decreasing.

A  $\frac{2}{15}$

B  $\frac{1}{5}$

C  $\frac{1}{3}$

D  $\frac{1}{2}$

E  $\frac{2}{3}$

F  $\frac{4}{5}$

G  $\frac{13}{15}$

**12** The minimum value of the function  $x^4 - p^2x^2$  is  $-9$

$p$  is a real number.

Find the minimum value of the function  $x^2 - px + 6$

**A**  $-3$

**B**  $6 - \frac{3\sqrt{2}}{2}$

**C**  $\frac{3}{2}$

**D**  $3$

**E**  $\frac{9}{2}$

**F**  $6 + \frac{3\sqrt{2}}{2}$

- 13** The function  $f$  is such that, for every integer  $n$

$$\int_n^{n+1} f(x) \, dx = n + 1$$

Evaluate

$$\sum_{r=1}^8 \left( \int_0^r f(x) \, dx \right)$$

- A** 36
- B** 84
- C** 120
- D** 165
- E** 204
- F** 288

14 *This question uses radians.*

Find the number of distinct values of  $x$  that satisfy the equation

$$(x + 1)(3 - x) = 2(1 - \cos(\pi x))$$

A 2

B 3

C 4

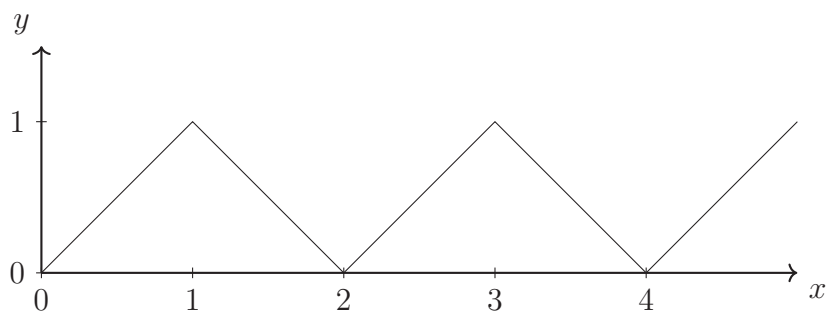
D 5

E 6

F 7



- 15 The diagram shows the graph of  $y = f(x)$ .



The graph consists of alternating straight-line segments of gradient 1 and  $-1$  and continues in this way for all values of  $x$ .

The function  $g$  is defined as

$$g(x) = \sum_{r=1}^{10} f(2^{r-1}x)$$

Find the value of

$$\int_0^1 g(x) \, dx$$

- A  $\frac{1023}{1024}$
- B  $\frac{1023}{512}$
- C 5
- D 10
- E  $\frac{55}{2}$
- F 55

16 Consider the expansion of

$$(a + bx)^n$$

The third term, in **ascending** powers of  $x$ , is  $105x^2$

The fourth term, in **ascending** powers of  $x$ , is  $210x^3$

The fourth term, in **descending** powers of  $x$ , is  $210x^3$

Find the value of  $\frac{a}{b}^2$

A  $\frac{1}{4}$

B  $\frac{4}{9}$

C  $\frac{25}{36}$

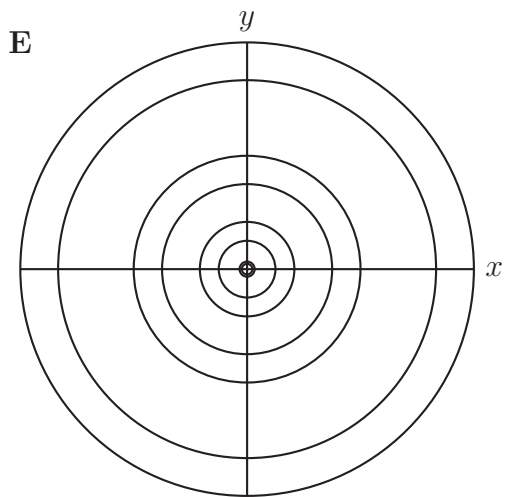
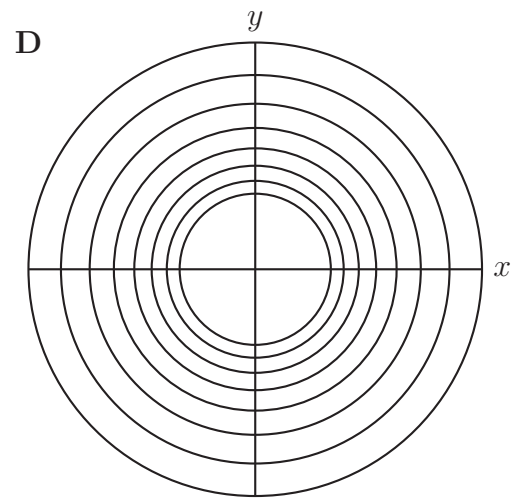
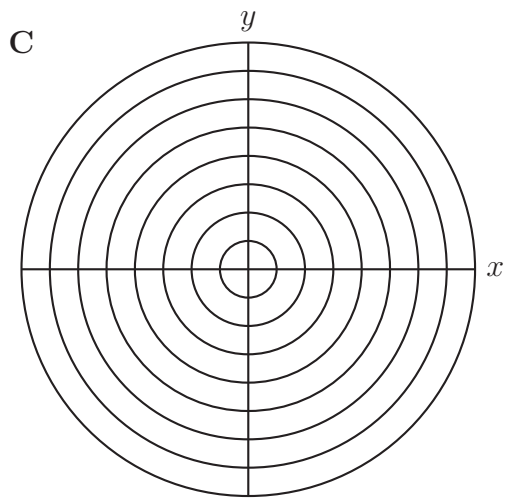
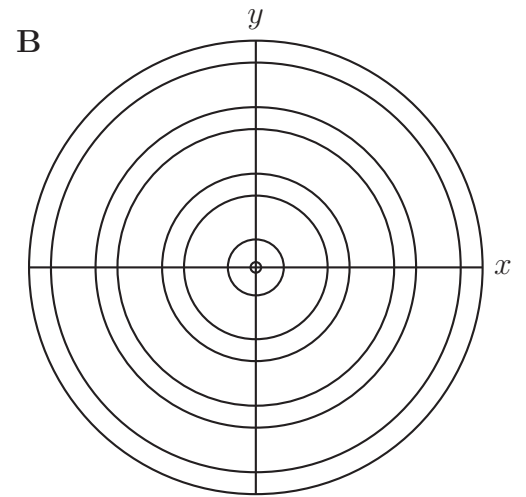
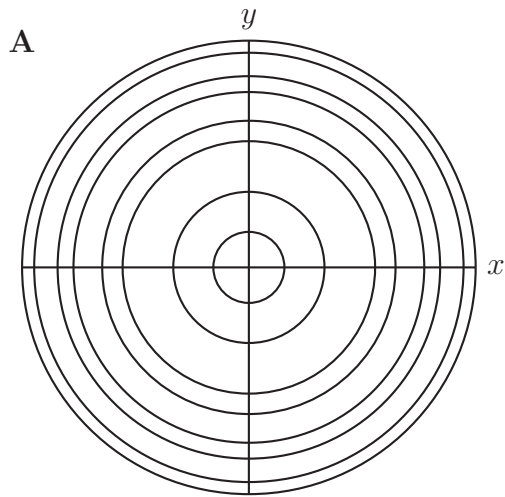
D  $\frac{5}{6}$

E 1

17 Which of the following sketches shows the graph of

$$\sin(x^2 + y^2) = \frac{1}{2}$$

where  $x^2 + y^2 \leq 8\pi$ ?



18 The curve with equation

$$x = y^2 - 6y + 11$$

is rotated  $90^\circ$  clockwise about the point  $P$  to give the curve  $C$ .

$P$  has  $x$ -coordinate  $-2$  and  $y$ -coordinate  $3$ .

What is the equation of  $C$  ?

**A**  $y = -x^2 - 4x - 3$

**B**  $y = -x^2 - 4x - 5$

**C**  $y = -x^2 - 6x - 7$

**D**  $y = -x^2 - 6x - 11$

**E**  $y = x^2 - 4x + 5$

**F**  $y = x^2 + 4x + 3$

**G**  $y = x^2 - 6x + 11$

**H**  $y = x^2 + 6x + 7$

19 The equation

$$\sin^2 (4^{\cos \theta} \times 60^\circ) = \frac{3}{4}$$

has exactly three solutions in the range  $0^\circ \leq \theta \leq x^\circ$

What is the range of all possible values of  $x$ ?

- A  $90 \leq x < 120$
- B  $90 \leq x < 270$
- C  $120 \leq x < 240$
- D  $270 \leq x < 300$
- E  $300 \leq x < 360$
- F  $450 \leq x < 630$

**20** Find the length of the curve with equation

$$2 \log_{10}(x - y) = \log_{10}(2 - 2x) + \log_{10}(y + 5)$$

- A** 5
- B** 10
- C** 15
- D**  $3\pi$
- E**  $9\pi$
- F**  $12\pi$

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